Proposition de sujet de thèse

Intitulé français du sujet de thèse proposé :
Structures non-autoadjointes et intégrables dans l'instabilité des résonances d'espace-temps de type trou noir.

Intitulé en anglais :
Non-selfadjoint and integrable structures in black hole spacetime resonance instability.

Unités de recherche :
Institut de Mathématiques de Bourgogne (IMB), Université de Bourgogne (Dijon, France)

(Possible co-direction à être considéré avec:
Instituto de Ciencias del Espacio, Consejo Superior de Investigaciones Cientificas (CSIC))

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Domaine scientifique principal de la thèse :
Integrability and scattering theory in general relativity

Domaine scientifique secondaire de la thèse :
Non-selfadjoint operators, infinite-dimensional Lie algebras

Description du projet scientifique
Antecedents: The adoption of a hyperboloidal approach in the setting of black hole scattering theory has permitted to cast black hole quasi-normal modes (or resonances) in terms of a non-selfadjoint spectral problem [1,2]. This has opened an avenue to the understanding of the instability of black hole quasi-normal mode (overtone) frequencies in terms of concepts and tools of the spectral theory of non-selfadjoint operators [3,4]. In particular, such black hole quasi-normal mode instability has been conjectured [5] to be accounted in terms of "degrees of freedom" located null infinity and controlled by spacetime asymptotic symmetries. On the other hand, the recent discovery of hidden (bulk) symmetries [6,7,8] in the setting black hole scattering, accounted in terms of Darboux transformations and conserved quantities of the Korteweg-de Vries (KdV) equation, offers the possibility of adopting an inverse scattering approach in which asymptotic and bulk symmetries are non-trivially coupled and the new degrees of freedom at infinity become additional scattering data for the inverse problem. Integrability notions would play a key role in these hidden structures.

Objective: The main goal of this thesis is to assess the role of infinite-dimensional Lie algebras in the construction of the enlarged phase of degrees of freedom potentially accounting for black hole quasi-normal mode instability. Specifically, i) a Lie algebra will be constructed coupling the bulk (hidden) symmetries coming from the background integrable theory with the asymptotic symmetries degrees at null infinity, and ii) a representation of the resulting Lie algebra will be constructed to account for the enlarged phase space. The ultimate goal is to show that these new degrees of freedom control black hole quasi-normal instability and explain the universality properties of such instability by means of an inverse scattering strategy.

Methodology: The methodology is based on the following elements:
i) *Hyperboloidal approach to scattering dynamics*. This framework permits to cast the evolution problem in terms of a non-selfadjoint infinitesimal time generator, in particular characterizing quasinormal modes in terms of the spectral problem of such non-selfadjoint operator and, on the other hand, identifying the explicit relation between non-selfadjointness with "outgoing fluxes" at null infinity. Hyperboloidal structures will play a key in the construction of the relevant infinitesimal Lie-algebras since, crucially, they couple the degrees of freedom in the bulk directly with those at the boundary through the mentioned "flux terms".
ii*) Inverse scattering, integrability and hidden symmetries*: an upgrade of the work by Lenzi and Sopuerta [6,7,8], developed in the setting of spacetime Cauchy slices, should be made to the setting of hyperboloidal slices. In particular, KdV structures will be cast in the hyperboloidal setting. An explicit relation between

quasi-normal modes and KdV constants will be explored and inverse scattering theory and related elements of integrability theory will be employed to reconstruct the bulk geometry from the scattering data at infinity.
iii) *Asymptotic spacetime symmetries*: the conjectured relation [5] between the adjoint of the infinitesimal time generator and the supertranslations in the BMS asymptotic symmetries will be assessed. In particular, Darboux transformations will be explored to account for the "flux part" of the non-selfadjoint infinitesimal time generator. The semi-direct action of the bulk hidden symmetries on asymptotic symmetries will be studied, in order to construct the complete infinite-dimensional Lie algebra and its appropriate (unitary) representation. This will lead to the notion of global spacetime "normal" (selfadjoint) modes for black holes.
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Connaissances et compétences requises :
Hyperboloidal formulation of scattering theory, general relativity, elements of integrability theory.

References:

[1] C. M. Warnick, *On Quasinormal Modes of Asymptotically Anti-de Sitter Black Holes*, Commun. Math. Phys. 333, 959 (2015).
[2] M. Ansorg and R. P. Macedo, Spectral Decomposition of Black-Hole Perturbations on Hyperboloidal Slices, Phys. Rev. D 93, 124016 (2016).
[3] Jaramillo J L, Macedo R P and Al Sheikh L, *Pseudospectrum and black hole quasinormal mode instability,* Phys. Rev. X 11 031003 (2021)
[4] Jaramillo J L, Macedo R P and Sheikh L A, *Gravitational wave signatures of black hole quasinormal mode instability*, Phys. Rev. Lett. 128 211102 (2022)
[5] Gasperín E and Jaramillo J L., *Energy scales and black hole pseudospectra: the structural role of the scalar product*, Class. Quantum Grav. 39 115010 (2022)
[6] M. Lenzi, C. F. Sopuerta, *Master functions and equations for perturbations of vacuum spherically symmetric spacetimes*, Physical Review D 104, 084053 (2021) DOI: 10.1103/PhysRevD.104.084053
[7] M. Lenzi, C. F. Sopuerta, *Darboux covariance: A hidden symmetry of perturbed Schwarzschild black holes*, Physical Review D 104, 124068 (2021) DOI: 10.1103/PhysRevD.104.124068
[8] M. Lenzi and C. F. Sopuerta, *BH greybody factors from Korteweg de Vries integrals: Theory*, Physical Review D 107, 044010 (2023) DOI: 10.1103/PhysRevD.107.044010 M. Lenzi and C. F. Sopuerta, *BH greybody factors from KdV integrals: Computation*, Phys. Rev. D 107, 084039 (2023) DOI: 10.1103/PhysRevD.107.084039