International Master in Mathematical Physics

– Math4Phys –

Guide 2023-24

Contents

1. Presentation

The Master in Mathematical Physics "Math4Phys" is a master course of study of the Department of Mathematics of the *Université de Bourgogne* (uB), which takes place at the *Institut de Mathématiques de Bourgogne* (IMB) in Dijon.

The main aim of the Master is to provide advanced lectures on the mathematical methods of modern theoretical physics in the framework of a mathematical curriculum.

Such an offer exists in France only in Dijon as the Mathematical Physics group of the IMB provides a unique environment for a program requiring a double competence in Mathematics and Physics.

The Mathematical Physics group of the IMB laboratory in Dijon is a unique research team in France with the ability to provide advanced lectures on the mathematical problems of modern physics.

2. Contacts

Official web page: `math4phys.ubfc.fr`

General enquiries: `math4phys@u-bourgogne.fr`

Address:

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Secretariat:

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Olga BUDKER – Applications – `olga.budker@u-bourgogne.fr`
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3. Application, registration and visa

All the details about the application procedure can be found at:
`www.ubfc.fr/en/formations/masters/`

For all practical aspects related to the admission letter, the student visa, and moving to France, one can refer to the following page:
`www.ubfc.fr/en/international/come-to-ubfc/practical-information/`

4. Scholarships

Each year the Master awards several scholarships of about 550 euros per month for up to 9 months to the most deserving students.

The number of the available scholarships depends on the secured funding.

All students accepted in M1 and M2 are automatically considered for the scholarships, which will be attributed solely on the basis of academic merit.

5. Calendar 2023/24

Applications begin	23/1/2023
Applications (nationals)	22/03
Applications end (nationals)	18/4
Applications end	23/6
Selection committee	5/7
Confirmation of participation*	1/9
First meeting M1	18/9 9:00
First meeting M2	18/9 16:00
Courses begin	19/9
Toussaint **	29/10 – 6/11
Courses end	22/12
Noël	23/12 – 7/1
Exams	8/1 – 12/1
Courses begin	15/1/2024
Hiver	24/2 – 4/3
Printemps	13/4 – 29/4
Courses end	3/5
Dissertation deadline	20/5
Exams, 1st session	13/5 – 17/5
Exams, 2nd session	17/6 – 28/6
Results	5/7

* Scholarship awardees are required to confirm their participation within two weeks of the offer of the scholarship.

** In *italics* the teaching breaks.

6. Courses M1

There are eight main courses plus a course of languages and a dissertation. Each course (apart from the language course) consists of 22 hours of lectures (CM) and 22 hours of exercise classes (TD). For French-speaking students the course FLE will be replaced by a course of English. For details about the dissertation see below.

	CM+TD (hrs)	ECTS
Differential geometry	22+22	7
Fourier analysis	22+22	7
Ordinary differential equations	22+22	7
Quantum physics	22+22	7
FLE (or English)	0+20	2
Groups and representations	22+22	6
Mathematical methods of classical mechanics	22+22	6
Numerical methods for physics	22+22	6
Partial differential equations	22+22	6
Dissertation		6

The schedule of the courses will be accessible via the ENT website of the *Université de Bourgogne* at the address: `ent.u-bourgogne.fr`

All courses are taught by members of the IMB in Dijon, unless otherwise noted. Other affiliations include the *Laboratoire de Mathématiques de Besançon* (LmB), the *Laboratoire Interdisciplinaire*

Carnot de Bourgogne (ICB) and the institute *Franche-Comté Electronique Mécanique Thermique et Optique – Sciences et Technologies* (FEMTO-ST).

Differential geometry (R. Uribe-Vargas, G. Dito)

Differentiable manifolds. Vector fields and flow-box theorem. Differential forms and Stokes' theorem. Tensors and vector bundles. Riemannian manifolds and connections. Geometry of gauge fields.

Fourier analysis (T. Daudé – LmB)

Fourier series for periodic functions of a real variable, Riemann–Lebesgue lemma, Dirichlet's conditions and Parseval's theorem. Convolution of two complex-valued functions defined on $\mathbb{R}^d$. Approximation to the identity. Fourier transform on R=L1 and Fourier inversion theorem. Plancherel theorem and Fourier transform on R=L2. Application to the resolution of some partial differential equations: Schrödinger equations, wave equations and heat equations.

Ordinary differential equations (U. Franz – LmB)

General existence and uniqueness theorems (Grönwall's Lemma, Picard-Lindelöf theorem, global existence, dependence on initial data). Linear differential systems (resolution of autonomous systems, systems with constant coefficients). Nonlinear autonomous systems (general properties of flows and orbits, phase portraits in two dimensions). Stability theorems (notions of stability, asymptotic stability, Lyapunov theorem).

Quantum physics (N. Kitanine, S. Leurent)

1. Introduction: 1.1 Observables in classical mechanics 1.2. Finite dimensional model of quantum mechanics 2. Basic principles of quantum mechanics: 2.1 States and observables in quantum mechanics 2.2 Quantum entanglement 2.3 Heisenberg uncertainty principle 2.4 Coordinate and momentum representations 2.6 Quantum dynamics: Schrödinger and Heisenberg pictures 2.5 Schrödinger equation 2.6 Classical limit 3. Quantum mechanics in one dimension: 3.1 Harmonic oscillator. Creation and annihilation operators 3.2 Scattering problem in one dimension 4. Quantum mechanics in 3D: 4.1 Free particle 4.2 Rotation group and angular momentum 4.3 Hydrogen atom 4.4 Spin 5. Multi-particle quantum systems, introduction.

Groups and representations (P. Schauenburg, S. Carrozza)

Notion of a group representation. Development of the structure theory for complex representations of finite groups: Theorems of Maschke and Schur. Tensor products and duality. Character theory. Induced representations. Some outlook beyond finite groups as time permits.

Mathematical methods of classical mechanics (G. Carlet, J. L. Jaramillo)

Lagrangian and Hamiltonian formalisms. Hamiltonian systems on symplectic manifolds. Variational principle and Hamilton-Jacobi equations. Poisson manifolds. Symmetries and momentum map.

Numerical methods for physics (N. Stoilov)

Interpolation and/or Linear systems. Numerical integration (classical rules, Gaussian quadrature rules). Fourier approximation. Numerical methods for solving ODE and PDE.

Partial differential equations (J. Lampart – ICB)

Distributions on $\mathbb{R}^n$: definition, convergence, distributions with compact support and tempered distributions, convolution, Fourier transform. Initial value problems: classical solutions, Fourier method, applications to the heat, wave and Schrödinger equations. Initial boundary value problems: heat operator on a bounded interval, variational formulation of the heat equation.

7. Courses M2

There are six main courses plus a course of languages and a dissertation. Each course (apart from the language course) consists of 36 hours of lectures (CM+TD) in the first semester and 30 hours in the second semester. For French-speaking students the course FLE will be replaced by a course of English. For details about the dissertation see below.

	CM+TD (hrs)	ECTS
Lie groups and Lie algebras	18+18	10
Mathematical methods of quantum physics	18+18	10
Riemann surfaces and integrable systems	18+18	10
FLE (or English)	0+20	2
Quantum field theory*	15+15	6
Quantum groups*	15+15	6
General relativity*	15+15	6
Introduction to TQFT*	15+15	6
Dissertation		10

* The students will have to choose 3 courses out of the 4 options available.

The schedule of the courses will be accessible via the ENT website of the *Université de Bourgogne* at the address: `ent.u-bourgogne.fr`

All courses are taught by members of the IMB.

Lie groups and Lie algebras (G. Dito)

1. Lie algebras: Basic definitions. Ideals and Lie subalgebras. Lie theorems. Real and complex forms. Universal enveloping algebra. Poincaré-Birkhoff-Witt theorem. Campbell-Hausdorff formula. 2. Structure of Lie algebras: Solvable, nilpotent and semisimple Lie algebras. Killing form. Lie and Engel theorems. Cartan criterion. Jordan decomposition. 3. Semisimple Lie algebras: Cartan subalgebra. Root system. Dynkin diagram. Classification of simple Lie algebras. Finite dimensional representations of sl(2).

Mathematical methods of quantum field theory (N. Kitanine)

1. Introduction, necessary background in mathematics and physics: distribution theory, functional analysis (spectral theorem), Lagrangian and Hamiltonian mechanics, basic principles of quantum mechanics, special relativity (Lorentz group and corresponding Lie algebra). 2. Classical field theory: Lagrangian formulation, conservation laws, Noether theorem for fields, examples (Klein-Gordon equation, sine-Gordon equation, non-linear Schrodinger equation, Maxwell equations etc.). Hamiltonian formulation, Poisson brackets. 3. Canonical quantisation of the free bosonic field: Weyl algebra, Fock space for bosons, lattice bosons, finite volume, infinite volume limit. Free bosons in 3+1 d. 4. Spinor representation of the Lorentz group and Dirac equation. Canonical quantization: Clifford algebra, Fock space for fermions: lattice fermions, finite volume, infinite volume limit. Free fermions in 3+1 d. 5. Introduction to the interacting field theories: Interaction picture, Dyson formula, Wick theorem, Feynman diagrams. 6. (if there is time) Introduction to quantum integrability.

Riemann surfaces and integrable systems (G. Carlet)

Definition of Riemann surface and basic examples, plane algebraic curves, hyperelliptic curves, holomorphic coverings, fundamental group, Riemann-Hurwitz theorem, homology groups, sheaves, Cech and sheaf cohomology, meromorphic functions, abelian differentials, integration theorems, divisors, Abel-Jacobi map, Abel theorem, Riemann-Roch theorem, Serre duality.

Quantum field theory (T. Kimura)

Lagrangian formalism and symmetry. Path integral formalism. Interacting fields and perturbation theory. Loop correction and renormalization. Quantization of non-Abelian gauge theory. Spontaneous symmetry breaking.

Quantum groups (P. Schauenberg)

General relativity (J. L. Jaramillo)

Introduction to TQFT (L. Woike, R. Detcherry)

Part I (R.D.): 1. Axiomatic description of topological field theories symmetric monoidal categories, bordisms, Atiyah's definition of a topological field theory, morphisms of topological field theories, the one-dimensional case, dimensional reduction, universal construction. 2. Two-dimensional topological field theories presentation of the two-dimensional bordism category in terms of generators and relations, commutative Frobenius algebras, the classification theorem, examples and applications, e.g. Mednykh's formula.
Part II (L.W): 3. Introduction to three-dimensional topological field theories via Dijkgraaf-Witten theories finite path integrals and linearization of groupoids, geometric construction of Dijkgraaf-Witten theories via principal fiber bundles, relation to Drinfeld doubles, outlook on the Reshetikhin-Turaev construction (surgery) and the classification of once-extended three-dimensional topological field theories by semisimple modular categories, the framing anomaly.

8. Dissertation M1

The students are required to choose a supervisor and a topic during the month of October and to work on the project under the guidance of the supervisor during the whole academic year. Depending on the actual number of students the project might have to be done in pairs of students.

The work done should be summarised in a dissertation, to be submitted in May.

The general rules for the dissertation are the following:

1. a maximum of 25 pages in TeX,
2. it should not be a direct rephrasing of some chapter of a book,
3. it should contain some personal and detailed take on a specific proof or computation.

Extra material can be included in the appendix without limitation on the number of pages, but won't be evaluated.

The dissertation has to be submitted before the deadline to one of the coordinators and will be evaluated by the supervisor.

The guidelines for evaluation are the following: understanding of the material, quality of the writing, originality in the treatment of the topic, engagement of the student.

The assessment of the supervisor takes into account the work done by the student during the year and the quality of the dissertation.

All the reports will be scanned by an anti-plagiarism software. In the case of evident plagiarism the mark will be zero.

It is not possible to retake the evaluation of the dissertation in the second session.

9. Dissertation M2

The students are required to choose a supervisor and a topic during the month of October and to work on the project under the guidance of the supervisor during the whole academic year. For more details please contact the M2 coordinator.

10. Exams

The exams of the first semester take place in January, those of the second semester in May, see the calendar above.

Typically the exam is a written test which lasts two or three hours.

For the course "Numerical methods for physics" there is also a partial examination during the semester which counts towards 1/3 of the total grade.

Marks and compensation. Possible outcomes of an exam are a mark between 0 and 20 or *DEF* (*défaillant*). A course is passed if the mark is greater or equal than 10. *DEF* is attributed to a student that is absent at the final exam, or to a student that is absent at the partial exam and does not have an official justification. Students that are absent at the partial exam but have an official justification will be given a zero mark, unless an extra session is organised, at the discretion of the teacher.

If a student during a semester has one or more exams with a mark less than 10 but the average of the marks of that semester is greater or equal to 10, then those exams are validated "by compensation". Exams are also validated by compensation if the average of the marks of the year is greater or equal to 10. The marks of the exams validated by compensation cannot be improved by retaking the exams later.

If an exam is marked *DEF* then no course in that semester can be validated by compensation.

Exams that are not passed at the first attempt and are not validated by compensation can be retaken in the second session in June.

Official rules. The general rules for the exams are detailed in the official document "Référentiel commun des études – Université de Bourgogne", available here:
`www.u-bourgogne.fr/images/stories/odf/ODF-referentiel-etudes-lmd.pdf`

The specific rules for the exams of the Master 1 are detailed in the "Fiche filiére", available here:
`www.u-bourgogne.fr/images/stories/odf/master/ff-mathematical-physics-m1.pdf`

11. Repeating the M1

Because the Master receives several outstanding applications each year and the number of available places is limited, students that fail the first year in general will not be allowed to repeat it. Only exceptional circumstances will be considered.

Students that have failed the second session of exams in June and wish to repeat the year should address their application to the coordinators within one week of the publication of the results of the second session.

The students readmitted to the M1 will not benefit from a scholarship.

12. F.A.Q.

Q: How many recommendation letters should I attach to my application? Are they compulsory?
A: Recommendation letters are not required, but they can be attached to the application or sent to `math4phys@u-bourgogne.fr` if one wishes so.

Q: I have been accepted to the master, what should I do?
A: You will receive instructions from the administration about the procedure to register and to other practical matter. ?

Q: When should I confirm my participation to the master?
A: We suggest you confirm your participation to (or your withdrawal from) the master as soon as possible, to quickly advance the registration procedures. The formal deadline is in the beginning of September (see the calendar in this Guide). If you are offered a scholarship we require that you confirm your participation within two weeks of the offer.

Q: At which stage can I apply for a scholarship?
A: You will not need to apply specifically for a scholarship, if you are accepted at the Master you will be automatically considered for the scholarship.

Q: How many students will be funded?
A: There number of available scholarships depends on funding that is subject to changes every year.

Q: What are the requirements for the scholarships?
A: The selection is based on academic merit, no other criteria are considered.

Q: What is included in a scholarship?
A: The scholarships are around 550 euros per month. Tuition fees are not included.

Q: What is an estimate of the cost of life for a student in Dijon?
A: As an estimate, lodging is around 200 euros/month. Tuition fee is around 240 euros/yr and there is an extra compulsory contribution for 'student life' of 95 euros/yr. See also:
`www.ubfc.fr/en/international/come-to-ubfc/practical-information/`

Q: When will scholarship recipients be announced?
A: Depending on the type of scholarship (we have different sources of financing), recipients will be announced in the months of July-August-September.

Q: Where can I find other sources of scholarships?
A: Unfortunately we are not aware of other sources of funding. We advise international students to enquire at their own institutions for exchanges programmes with France or other funding to study abroad.

Q: Is it possible to work part-time while following the master?
Since the schedule of courses is very tight, we do not advise to work alongside the master.

(updated 10/7/2023)