PhD position in Dijon (France) with a 6-month stay in Vancouver (Canada)

- **Period:** From October 1, 2022 to September 30, 2025.
- **Supervision:**
  - Supervisor: Ronan Terpereau, Institut de Mathématiques de Bourgogne (UMR 5584) , Université Bourgogne Franche-Comté.
  - Co-Supervisor: Lucy Moser-Jauslin, Institut de Mathématiques de Bourgogne (UMR 5584), Université Bourgogne Franche-Comté.
  - Scientific referent in PIMS : Zinovy Reichstein, professor of Mathematics at University of British Columbia, member of PIMS-CNRS International Research Lab.
- **Scientific domain:** Algebraic geometry, theory of algebraic groups, Lie theory.
- **Title:** Complexity-one varieties over perfect fields.
- **Description of the project:** There is a well-established combinatorial description for algebraic varieties of complexity ≤ 1 over an algebraically closed field (see [4, 3, 7]). The complexity-zero case, corresponding to *spherical varieties*, was extended recently to arbitrary perfect base fields (see [2, 8, 6, 1]). The main goal of this PhD thesis will be to extend the combinatorial description of complexity-one varieties to an arbitrary perfect base field k_0 (for instance $k_0 = \mathbb{R}$ or $\mathbb{Q}_p$).

  As a first step, the student will consider the case of almost homogeneous SL_2-threefolds, studied in [5, § 3] when $k_0 = \mathbb{R}$, and the case of SL_3/T and its completions, with T a maximal torus. In both cases, the combinatorial data over an algebraically closed field are described in [7, § 16], but the main point will be to determine how to encode the Galois descent data compatibly. As a second step, the student will tackle the general complexity-one case, based on the results obtained previously. Then, as a third step, he/she will apply this description to determine k_0-forms of classical families of complexity-one varieties (e.g. one-parameter families of spherical varieties or certain Fano varieties) and study rationality questions for these k_0-forms (even for real threefolds, very little is known in this direction). It is especially during this third step that Prof. Reichstein's role in this project will be crucial since he is an expert of algebraic groups over non-closed fields and has worked on rationality problems in algebraic geometry.
- **Some video links:** To know more about this subject, you can watch a recorded survey talk on this research work that I gave at the "Zoom Algebraic Geometry" Seminar:
  `https://www.youtube.com/watch?v=G4cMnN9pvio`
  or the recorded talk given by Lucy on our research work at the "Quadratic forms, Linear algebraic groups and Beyond" Online Seminar:
  `https://drive.google.com/file/d/10uewCnrsndk9Fnu7fV67CVg_C5GRTblB/view`
- **Profile:** The applicant must have a Master's degree in fundamental mathematics, and he/she expected to have taken courses in algebraic geometry, algebraic groups theory or Lie theory.

  The hired PhD student will be a member of the team "Géométrie, Algèbre, Dyamique et Topologie" of the lab *Institut de Mathématiques de Bourgogne*, and he/she will be expected to participate in the scientific activities of the lab.
- **Stay at PIMS:** The PhD student will visit UBC during the Spring semester 2024 (from January to June 2024). During his/her research stay at UBC, the student will be part of the research

group of Prof. Reichstein, and he/she will also have the opportunity to interact with other members of the Mathematics department.

- **To apply:** You can send me your application by e-mail (`ronan.terpereau@u-bourgogne.fr`); it must include a CV, a transcript of your grades, a cover letter, and a list of contacts among your teachers. Also, do not hesitate to contact me or Lucy if you have questions.

  The deadline to apply is May 31.

- **References:**

[1] M. Borovoi and G. Gagliardi. Existence of Equivariant Models of Spherical Varieties and Other G-varieties. *International Mathematics Research Notices*, 07 2021.

[2] M. Huruguen. Toric varieties and spherical embeddings over an arbitrary field. *J. Algebra*, 342:212–234, 2011.

[3] F. Knop. The Luna-Vust theory of spherical embeddings. In *Proceedings of the Hyderabad Conference on Algebraic Groups (Hyderabad, 1989)*, pages 225–249. Manoj Prakashan, Madras, 1991.

[4] D. Luna and T. Vust. Plongements d'espaces homogènes. *Comment. Math. Helv.*, 58(2):186–245, 1983.

[5] L. Moser-Jauslin and R. Terpereau. Forms of almost homogeneous varieties over perfect fields. *arXiv:2008.05197*.

[6] L. Moser-Jauslin and R. Terpereau. Real Structures on Horospherical Varieties (with an appendix by M. Borovoi). *Michigan Mathematical Journal*, (Advance Publication):1 – 38, 2021.

[7] D. A. Timashev. *Homogeneous spaces and equivariant embeddings*, volume 138 of *Encyclopaedia of Mathematical Sciences*. Springer, Heidelberg, 2011. Invariant Theory and Algebraic Transformation Groups, 8.

[8] T. Wedhorn. Spherical spaces. *Ann. Inst. Fourier (Grenoble)*, 68(1):229–256, 2018.