

Ecole Doctorale Carnot-Pasteur : Proposition de sujet de thèse

Intitulé français du sujet de thèse proposé :

Catégories modulaires groupe-théoriques

Intitulé en anglais :

Group theoretical modular categories

Unité de recherche : Institut de mathématiques de Bourgogne UMR 5584 du CNRS

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Domaine scientifique principal de la thèse : Mathématiques, mathématique physique

Description du projet scientifique :

I will start with a very specific problem : The higher Frobenius-Schur indicators of the representations of the Drinfeld double of a symmetric group are known to be integers. They are nonnegative for symmetric groups up to degree 25 ; this, however is an experimental finding [1,3] based on heavy computer calculations. For symmetric groups themselves (not their doubles) the result was established by Scharf [2]. It is a combinatorial fact on the symmetric groups (the root-number counting function is a character), and was later found to be related to the theory of Lie characters of symmetric groups [4].

The problem just described would be exciting to solve (and could well prove too elusive), but it also serves as a particular exhibit for the larger field of problems in which the thesis should be situated. To wit, fusion categories (semisimple categories with a tensor product, such as representations of groups or certain quantum groups) and modular fusion categories are linked to many topics in mathematical physics, in particular giving rise to a machinery of algebraically defined topological invariants known as topological quantum field theory ; finite groups in turn give rise to very specific such categories. Both constructions are well established, but the structure theory and classification of fusion and modular categories are areas of active investigation ; while the specific examples coming from finite groups are perhaps the most easily constructed, there is still a lot of room to exploit specifics from group (representation) theory in this context. For the specific problem described above this would likely involve the theory of Lie characters or a generalization thereof ; a solution would shed light on the topological quantum field theories associated to symmetric groups.

[1] R. Courter. Computing higher indicators for the double of a symmetric group. 2012. Thesis (Ph.D.)—University of Southern California.

[2] T. Scharf. Die Wurzelanzahlfunktion in symmetrischen Gruppen. J. Algebra, 139(2):446–457, 1991.

[3] P. Schauenburg. Higher Frobenius-Schur indicators for Drinfeld doubles of finite groups through characters of centralizers. ArXiv e-prints, Apr. 2016.

[4] M. Schocker. On the root symmetry of higher Lie characters. Arch. Math. (Basel), 80(4):337–346, 2003.

Connaissances et compétences requises :

Algèbre : Algèbre linéaire approfondie, notions de la théorie des groupes et représentations, des anneaux et modules.