

CLUSE MATHEMATICAL SCHOOL
GROUP THEORY
Messigny et Vantoux
October 28th to november 1st, 2007
ABSTRACTS

1 Mini-courses

Goulnara ARJANTSEVA (Université de Genève, Switzerland)

Title: *Random groups.*

Abstract: A random group is a generic representative of a given class of groups. Studying of such a group leads to a uniform vision on groups in general and has spectacular applications. This point of view is due to Gromov. The objective of this course is to provide participants with understanding of modern viewpoint on random groups. We will discuss different models of random groups, coming from combinatorial, statistical, and topological approaches. All of these models lead to the same conclusion that a generic (finitely presented) group is hyperbolic.

Laurent BARTHOLDI (Ecole polytechnique Fédérale de Lausanne, Switzerland)

Title: *Lie algebras associated to discrete groups.*

Abstract: A construction due to Magnus (in the 30's) associates a Lie algebra with an arbitrary group, as follows. The lower central series of the group gives it a descending filtration, whose associated graded is naturally a Lie algebra.

This algebra is, in many respects, a much simpler object than the group; yet it still captures some essential elements of the group's structure.

In this course, I will

1. detail the basics of this construction, and its connection to the Magnus embedding
2. discuss the work by Lazard on this Lie algebra's structure in the case of "analytic" groups, such as matrix groups

3. describe the interplay between the group and the Lie algebra for some 1-relator, or "small cancellation" groups, following work by Labute
4. explain related results on braid groups and automorphism groups of free groups

Cédric BONNAFE (Université de Besançon, France)

Title: *Polynomial invariants of finite groups, and reflection groups.*

Abstract: The first part of the course will be devoted to the study of polynomial invariants by a finite group : our goal is to give a proof of Shephard-Todd-Chevalley Theorem that says that the ring of invariant under a finite group G is a polynomial ring if and only if the group G is generated by reflections.

The second part is devoted to the further study of the connection between the structure of finite reflection groups and the structure of the ring of polynomial invariants.

If time is left, we will have a look at diagonal invariants of finite reflection groups and topics related with the $n!$ -conjecture.

2 30 minutes lectures

Yago ANTOLIN PICHEL (Universitat Autònoma de Barcelona, Spain)

Title: *Local indicability and one-relator products.*

Abstract: A group is local indicable if every finitely generated subgroup has a infinite cyclic quotient. Many classical results of one-relator groups, such as Freiheitssatz or Lyndon's identity theorem, generalizes to one-relator products of locally indicable groups. I will describe a technique that gives a shorter proof of a theorem due to Jim Howie: torsion-free one-relator products of locally indicable groups are locally indicable. This is a joint work with Warren Dicks.

Vincent BECK (Université Paris 7, France)

Title: *Relative invariants of complex reflection groups.*

Abstract: Let G be a complex reflection groups, χ a linear character and M a G -module. We construct and study an exterior algebra structure on the χ -invariant of the tensor product of the symmetric algebra of V^* (where V is the vector space on which G acts naturally) by the exterior algebra of M^* .

Paolo BELLINGERI (Università di Milano - Bicocca, Italy)

Title: *Residual properties of topological generalisation of (pure) braid groups.*

Abstract: This talk deals with combinatorial properties of topological generalisations of braid groups, in particular, their lower central series and their related residual properties. In particular we determine lower central series for (pure) braid groups of orientable surfaces (GT/07054201 and GT/0512155) and we study some combinatorial properties

of a framed version of pure braid groups of closed surfaces (current work with Sylvain Gervais).

Fabrice CASTEL (Université de Bourgogne, France)

Title: *Lien entre les groupes d'automorphismes des groupes de tresses et des mapping class groups des surfaces.*

Abstract: Après avoir défini le groupe de tresses et le mapping class group d'une surface, on énoncera un théorème décrivant l'ensemble des morphismes du groupe de tresses dans le mapping class group de surfaces convenables. On verra comment en découlent le groupe d'automorphismes du groupe de tresses et celui du mapping class group des surfaces.

Maria CHLOUVERAKI (EPFL, Switzerland)

Title: *Essential hyperplanes of complex reflection groups.*

Abstract: Following the definition of Rouquier for the "families of characters" of a Weyl group and its generalization to the case of a complex reflection group W , already used in the works of Broué-Kim and Malle-Rouquier, we show that the Rouquier blocks of a cyclotomic Hecke algebra associated with W depend only on numerical data defined by the generic Hecke algebra : the essential hyperplanes of W .

Laura CIOBANU (Centre de Recerca Matemàtica, Spain)

Title: *Equations in tree-free groups*

Abstract: In this talk I will focus on an often studied equation, $x^p y^q z^r = 1$, in tree-free groups. Tree-free groups are groups which act freely and without inversions by isometries on some Lambda-tree, where Lambda is an ordered abelian group. Let G be a tree-free group and let x, y, z be elements in G . We show that if $x^p y^q z^r = 1$ with integers p, q, r at least 4, then x, y and z commute. As a result, the one-relator groups with $x^p y^q z^r = 1$ as relator, are examples of hyperbolic and CAT(-1) groups which do not act freely on any Lambda-tree. This is joint work with N. Brady, A. Martino and S. O'Rourke.

Jean-Baptiste GRAMAIN (University of Aberdeen, UK)

Title : *Generalized blocks of unipotent characters in $GL(n, q)$.*

Abstract : In a paper published in 2003, B. Külshammer, J. B. Olsson and G. R. Robinson have generalized Brauer's notion of blocks of irreducible characters of a finite group, and studied the case of the symmetric group. In this talk, we present some aspects of the theory of blocks and its generalization, as well as a generalized Nakayama Conjecture. We then give a construction of blocks of unipotent characters in the finite general linear group, and study an analogue of the Nakayama Conjecture.

Ivan MARIN (Université Paris 7, France)

Title: *Lie algebras associated to reflection groups.*

Abstract: I will present finite-dimensional, reductive Lie algebras which are naturally associated with finite reflection groups. The most interesting one can be interpreted as the

Lie algebra of the Zariski closure of the corresponding braid group inside the corresponding Hecke algebra, and thus deserves the name of "infinitesimal Hecke algebra".

Mattia PERRONE (LATP, Marseille, France)

Title: *Haagerup inequality for radial functions on uniform lattices in Lie groups.*

Abstract: We show that radial RD property is preserved when we restrict to cocompact lattice subgroups Γ of a given locally compact group G . We classify all lattices in Lie groups with RRD property w.r.t. a suitable length function L . In particular this give a new criterion to establish cocompactness of lattices in semisimple Lie groups.

Simona SETTEPANELLA (Pisa University, Italy)

Title: *Combinatorial Morse theory and minimality of hyperplane arrangements.*

Abstract: In this talk we show how to prove the minimality of configuration space by using the combinatorial Morse Theory.