

À quel bandit faire confiance pour exécuter un ordre (de bourse) ?

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Journées Algorithmes stochastiques

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Two armed bandit algorithm

- ▷ **HISTORY.** • Goes back to Mathematical Psychology: M.F. Norman, [1968] On linear Models with Two Absorbing Barriers, *J. of Mathematical Psychology*, **5**, pp.225-241..
- Learning Automata: K.S. Narendra, M.A.L. Thathachar [1974] Learning Automata - A survey, *IEEE Trans. Systems, Man., Cybernetics*, S.M.C-4, pp. 323-334. learning automata. Known as “LRI” for “Linear Reward-Inaction” algorithm.

▷ **PRINCIPLE (ADAPTED TO HEDGE FUND MANAGEMENT):**

- Two traders A and B manage X_n % and $1 - X_n$ % of a fund at time n .
- Evaluation at random, proportionally to the managed share \equiv head & tail with parameter $X_n \in (0, 1)$.
- Respective performances at time n are rated (if evaluated) by a 0-1 principle:

$$A_n = \{A \text{ is OK at time } n\}, \quad B_n = \{B \text{ is OK at time } n\}$$

- Reward is proportional to the other's trader managed share.

- ▷ Focus on the share managed by trader A .

$$X_{n+1} = X_n + \gamma_{n+1} \times \begin{cases} \mathbf{1}_{\{U_{n+1} \leq X_n\} \cap A_{n+1}} (1 - X_n) \\ \mathbf{1}_{\{U_{n+1} > X_n\} \cap B_{n+1}} (-X_n) \end{cases} \in (0, 1)$$

where $X_0 = x \in (0, 1]$ and

- $(U_n)_{n \geq 1}$ is iid, $U_1 \sim U([0, 1])$, independent of
- $(A_n, B_n)_{n \geq 1}$, iid, $\mathbb{P}(A_n) = p_A > \mathbb{P}(B_n) = p_B$.
- $\gamma_n \in (0, 1]$ a decreasing “gain parameter” such that

$$\sum_{n \geq 1} \gamma_n = +\infty \quad \text{and} \quad \sum_{n \geq 1} \gamma_n^2 < +\infty$$

- $\mathcal{F}_n = \sigma(U_k, A_k, B_k, 1 \leq k \leq n)$ the filtration of the procedure.

Canonical form

▷ Set $\pi = p_A - p_B > 0$.

$$h(x) = \mathbb{E} \left(\frac{\Delta X_{n+1}}{\gamma_{n+1}} \mid X_n = x \right) = \pi X_n (1 - X_n)$$

▷ Canonical form:

$$X_{n+1} = X_n + \gamma_{n+1} \pi X_n (1 - X_n) + \Delta M_{n+1}$$

where

$$\Delta M_{n+1} = (1 - X_n) \mathbf{1}_{\{U_{n+1} \leq X_n\} \cap A_{n+1}} - X_n \mathbf{1}_{\{U_{n+1} > X_n\} \cap B_{n+1}} - \pi X_n (1 - X_n)$$

and

$$\mathbb{E}(\Delta M_{n+1}^2 \mid \mathcal{F}_n) = X_n (1 - X_n) (p_A (1 - X_n) + p_B X_n - \pi^2 X_n (1 - X_n))$$

▷ $(X_n)_{n \geq 1}$ is a bounded sub-martingale hence $X_n \xrightarrow{a.s. \& L^1} X_\infty$
so that

$$0 \leq \mathbb{E} \left(\sum_{n \geq 1} \gamma_n X_n (1 - X_n) \right) = \frac{1}{\pi} (\mathbb{E}(X_n) - x) \longrightarrow \frac{1}{\pi} (\mathbb{E}X_\infty - x)$$

hence

$$\sum_{n \geq 1} \gamma_n X_n (1 - X_n) < +\infty \quad a.s.$$

so that

$$X_\infty(1 - X_\infty) = \liminf_n X_n(1 - X_n) = 0 \quad a.s.$$

i.e.

$$\mathbb{P}(X_\infty = 0 \text{ or } 1) = 1.$$

▷ But...

0 is a (noiseless!) trap and 1 is a (noiseless!) target.

An old problem

▷ The ODE method (fails):

$$\dot{x} = \pi x(1 - x)$$

then

$$\forall x_0 \in (0, 1], \quad \Phi(x_0, t) = x_0 \frac{x_0 e^{\pi t}}{1 - x_0 + x_0 e^{\pi t}} \longrightarrow 1 \quad \text{as } t \rightarrow \infty.$$

1 is an attractor (target) with attracting area $A_1 = (0, 1]$.

Old but celebrated Kushner-Clark's Theorem says:

“If X_n visits infinitely often a compact subset of A_1 then $X_n \rightarrow 1 \dots$ ”

i.e.

$$\limsup_n X_n > 0 \implies \lim_n X_n = 1 \quad \text{which is... empty.}$$

▷ “Anti-trap” Theorems (Pemantle [1990], Lazarev [1992], Brandière-Duflo [1996], etc) fail as well.

Typically (in 1-dimension)

$$X_{n+1} = X_n + \gamma_{n+1}H(X_n, U_{n+1}), \quad (U_n)_{n \geq 1} \text{ i.i.d.}, \sim \mathcal{L}(U),$$

with standard decreasing step assumption. If

$$h(x) = \mathbb{E}(H(x, U)), \quad h(x^*) = 0, \quad h'(x^*) > 0 \quad \text{and} \quad \mathbb{E} H^2(x^*, U) > 0,$$

then $\mathbb{P}(X_n \rightarrow x^*) = 0$.

▷ Adding “repulsive” noise at the trap 0 is impossible: 0 is an endpoint of the state space...

▷ Question:

When can the bandit be trusted (to manage a hedge fund)?

(Like *Olympia Capital Management*, see M. Niang [2003]).

When can the bandit be trusted?

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▷ An answer by Lambertson-P.-Tarrès (2005) in *AAP*.

Let $\gamma_n = \left(\frac{C}{C+n}\right)^\alpha$, $0 < \alpha \leq 1$, $C > 0$ (for the talk).

- If $0 < \alpha < 1$ or $\alpha = 1$ and $C > \frac{1}{p_B}$ then the bandit is **fallible**:

$$\mathbb{P}(X_\infty = 0) > 0.$$

- If $\alpha = 1$ and $C \leq \frac{1}{p_B}$, the bandit is **infallible** (e.g. if $C = 1$).

What about the rate of convergence?

What about the rate of convergence?

Lamberton-P. (2007) in *Stochastic Analysis and Applications*.

- ▶ Usual theorems fail: CLT at rate $\sqrt{\gamma_n}$, LLI, Ruppert-Poliak principle, etc.)
- ▶ Beginning of completely new story...

FIRST STEP.

$$\{X_n \rightarrow 0\} = \left\{ \sum_n X_n < +\infty \right\}$$

▷ Set $\tilde{X}_n = \frac{X_n}{\gamma_n}$, $n \geq 0$ ($\gamma_n = 1$).

$$\tilde{X}_{n+1} = \tilde{X}_n + X_n \left(\frac{1}{\gamma_{n+1}} - \frac{1}{\gamma_n} + \pi(1 - X_n) \right) + \Delta M_{n+1}$$

$$= X_0 + \sum_{k=1}^{n+1} X_{k-1} \left(\frac{1}{C} + \pi(1 - X_{k-1}) \right) + M_{n+1}$$

$$\frac{X_{n+1}}{\gamma_{n+1}} \geq x + \left(\frac{1}{C} + \pi \right) \sum_{k=0}^n X_k(1 - X_k) + M_{n+1}.$$

▷ Set $S_n = x + X_1 + \cdots + X_n$,

On $\{X_n \rightarrow 0\} \cap \{\sum_n X_n = +\infty\}$, $\langle M \rangle_n \sim p_A S_{n-1}$

hence

$$\liminf_n \frac{X_n}{\gamma_n S_{n-1}} \geq 0 + \frac{1}{C} + \pi + \lim_n \frac{M_n}{\langle M \rangle_n} = \frac{1}{C} + \pi.$$

so that, for $n \geq n_0$,

$$\frac{\Delta S_n}{S_{n-1}} \geq \left(\frac{1}{C} + \frac{\pi}{2} \right) \underbrace{\frac{C}{C+n}}_{=\gamma_n}$$

i.e.

$$S_n \geq K n^{1+\frac{C\pi}{2}}$$

which in turn implies

$$X_n \geq K' \frac{n^{1+\frac{C\pi}{2}}}{n+C} \sim K' n^{\frac{C\pi}{2}} \longrightarrow +\infty!$$

Geometric convergence to the trap !

▷ Starting from the compensator identity (using (U_n) i.i.d.)

$$\left\{ \sum_n X_n < +\infty \right\} = \left\{ \sum_n \mathbf{1}_{\{U_{n+1} \leq X_n\}} < +\infty \right\}$$

on the event $\{X_n \rightarrow 0\}$, $X_n < U_{n+1}$ for $n \geq n_0$ (A is never tested!...).

▷ Consequently, on $\{X_n \rightarrow 0\}$,

$$\begin{aligned} X_n &= X_{n_0} \prod_{k=n_0+1}^n (1 - \gamma_k \mathbf{1}_{B_k}) \\ &\sim \Xi_x \prod_{k=1}^n (1 - p_B \gamma_k) \\ &\sim \Xi'_x \times \frac{1}{n^{C p_B}}. \end{aligned}$$

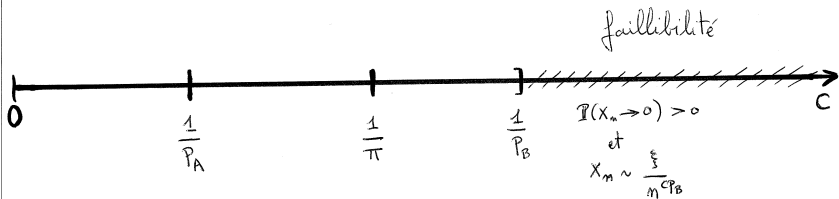
▷ **First conclusion for the trap 0:** one retrieves

- If $Cp_B \leq 1$, then **infallibility!** ($\forall x \in (0, 1]$)
- If $Cp_B > 1$, then **fallibility with positive probability:**

$$\begin{aligned}\mathbb{P}(X_n \rightarrow 0) &= \mathbb{P}\left(\sum_n X_n < +\infty\right) = \mathbb{P}\left(\sum_n \{U_{n+1} \leq X_n\} < +\infty\right) \\ &= \lim_n \mathbb{P}\left(\bigcap_{k \geq n} \{U_{k+1} > X_k\}\right) \quad (A \text{ no longer tested}) \\ &= \lim_n \mathbb{P}\left(\bigcap_{k \geq n} \{U_{k+1} > X_n \prod_{\ell=n}^k (1 - \gamma_\ell \mathbf{1}_{B_\ell})\}\right) \\ &= \lim_n \mathbb{E}\left(\mathbb{P}\left(\bigcap_{k \geq n} \{U_{k+1} > X_n \prod_{\ell=n}^k (1 - \gamma_\ell \mathbf{1}_{B_\ell})\} \mid \sigma(X_n, B_\ell, \ell \geq n)\right)\right)\end{aligned}$$

$$\begin{aligned}
\mathbb{P}(X_\infty = 0) &= \lim_n \mathbb{E} \left(\prod_{k \geq n} \left(1 - X_n \prod_{\ell=n}^k (1 - \gamma_\ell p_B) \right) \right) \\
&\geq \lim_n \mathbb{E} \left(\prod_{k \geq n} \left(1 - \prod_{\ell=n}^k \left(1 - \frac{C p_B}{C + \ell} \right) \right) \right) \\
&> 0 \quad \text{if } C p_B > 1.
\end{aligned}$$

$$P_A > P_B \quad \text{et} \quad \gamma_m = \frac{C}{C+m}, \quad m \geq 1$$



$$\neq \emptyset \text{ si } P_A > P_B$$

And now toward the target...

▷ A symmetry argument. Set $\bar{X}_n = \frac{1-X_n}{\gamma_n}$.

$$\bar{X}_{n+1} = \bar{X}_n + (1 - X_n) \left(\frac{1}{C} - \pi X_n \right) - \Delta M_{n+1}$$

so that, if $\frac{1}{C} < \pi - \eta$, on the event $\{X_n \rightarrow 1\} \cap \{\sum_n 1 - X_n = +\infty\}$,

$$\limsup_n \frac{1 - X_n}{\gamma_n \bar{S}_n} \leq \frac{1}{C} - \pi + \eta < 0$$

Finally, one shows that... $1 - X_n \leq K_{\eta, C} n^{-(1+\eta C)}$!

$$\boxed{\text{If } C > \frac{1}{\pi}, \quad \{X_n \rightarrow 1\} = \left\{ \sum_n 1 - X_n < +\infty \right\}.}$$

Carrying over with the symmetry argument

▷ FIRST CONCLUSION FOR THE TARGET 1

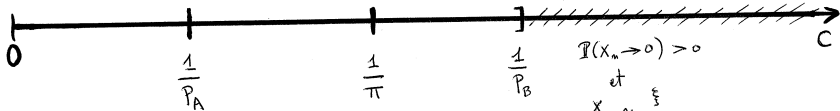
If $C > \frac{1}{\pi}$, then

$$\text{on } \{X_n \rightarrow 1\}, \quad X_n \sim \Xi'_x \times \frac{1}{n^{C_{p_A}}}.$$

$$P_A > P_B \quad \text{et} \quad \gamma_m = \frac{C}{C+m}, \quad m \geq 1$$

infaillibilité

faillibilité



convergence vers 1 à vitesse n^{-C/P_A} , $C/P_A > 1$

$\neq \emptyset$ si $P_A > 2P_B$

What about $C \leq \frac{1}{\pi}$?

▷ A NEW MARTINGALE.

$$1 - X_{n+1} = 1 - X_n(1 - \pi\gamma_{n+1}X_n) - \gamma_{n+1}\Delta_{n+1}$$

so that

$$Y_n := \frac{1 - X_n}{\prod_{1 \leq k \leq n} (1 - \pi\gamma_k X_{k-1})} = 1 - x - \sum_{k=1}^n \frac{\gamma_k}{\prod_{1 \leq \ell \leq k} (1 - \pi\gamma_\ell X_{\ell-1})} \Delta M_k$$

is a **new** non-negative **martingale**!

$$Y_n \longrightarrow Y_\infty \geq 0 \quad a.s., \quad Y_\infty \in L^1(\mathbb{P}).$$

Now, using $\sum_n \gamma_n(1 - X_{n-1}) < +\infty$ on $\{X_n \rightarrow 1\}$, one shows that

$$\xi := \prod_{n \geq 1} \left(\frac{1 - \pi\gamma_n X_{n-1}}{1 - \pi\gamma_n} \right) > 0 \quad \text{on} \quad \{X_n \rightarrow 1\}$$

since $\sum_n \gamma_n(1 - X_{n-1}) < +\infty$ on $\{X_n \rightarrow 1\}$.

▷ (Partial) Conclusion on the target 1

$$\frac{1 - X_n}{\prod_{1 \leq k \leq n} (1 - \pi \gamma_k)} \longrightarrow \xi Y_\infty \quad a.s.$$

i.e.

$$n^{C\pi}(1 - X_n) \longrightarrow \xi Y_\infty \geq 0 \quad a.s.$$

Is it the true rate ? *i.e.* $Y_\infty > 0$?

▷ A FUNDAMENTAL “TRAP INEQUALITY” (L.P.T.(2003))

Let $(Y_n)_{n \geq 0}$ be an L^2 -martingale with compensator $\langle Y \rangle_n$.

$$\mathbb{P}(Y_\infty = 0 | \mathcal{F}_n) \leq \frac{1}{Y_n^2} \mathbb{E}((Y_\infty - Y_n)^2 | \mathcal{F}_n) = \frac{\mathbb{E}(\langle Y \rangle_{n+1}^\infty | \mathcal{F}_n)}{Y_n^2}$$

▷ AND ITS APPLICATION...

$$\Delta \langle Y \rangle_n := \mathbb{E}((\Delta Y_n)^2 | \mathcal{F}_n) \leq \frac{\gamma_n^2}{\prod_{1 \leq k \leq n} (1 - \pi \gamma_k)^2} p_A X_{n-1} (1 - X_{n-1})$$

$$\leq \frac{\gamma_n^2}{\prod_{1 \leq k \leq n} (1 - \pi \gamma_k)} Y_{n-1}$$

$$\langle Y \rangle_n^\infty \leq \sum_{k \geq n+1} \frac{\gamma_k^2}{\prod_{1 \leq \ell \leq k} (1 - \pi \gamma_\ell)} Y_{k-1}$$

$$\mathbb{E}(\langle Y \rangle_n^\infty | \mathcal{F}_n) \leq Y_n \sum_{k \geq n+1} \frac{\gamma_k^2}{\prod_{1 \leq \ell \leq k} (1 - \pi \gamma_\ell)}$$

since $(Y_n)_{n \geq 0}$ is martingale.

$$\begin{aligned} \mathbb{P}(Y_\infty = 0 | \mathcal{F}_n) &\leq \frac{1}{Y_n} \sum_{k \geq n+1} \frac{\gamma_k^2}{\prod_{1 \leq \ell \leq k} (1 - \pi \gamma_\ell)} \\ &\lesssim \frac{1}{\xi} \frac{\prod_{1 \leq k \leq n} (1 - \pi \gamma_k)}{1 - X_n} \sum_{k \geq n+1} \frac{\gamma_k^2}{\prod_{1 \leq \ell \leq k} (1 - \pi \gamma_\ell)} \\ &\lesssim \frac{1}{\xi} \frac{n^{-\pi C}}{1 - X_n} \times n^{\pi C - 1} \\ &\lesssim \frac{1}{\xi} \frac{1}{n(1 - X_n)} \quad \text{owing to} \quad \gamma_n = \frac{C}{C+n}. \end{aligned}$$

The fastest way to 1 is to evaluate A at every instant n , so that

$$1 - X_n \geq (1-x) \prod_{k=1}^n (1 - \gamma_k \mathbf{1}_{A_k}) \sim (1-x) \xi' \prod_{k=1}^n (1 - p_A \gamma_k) \sim (1-x) \xi' n^{-C p_A}$$

so that

$$\mathbb{P}(Y_\infty = 0 \mid \mathcal{F}_n) \lesssim \xi'' \times \frac{1}{n^{1-C p_A}}.$$

▷ FIRST CONCLUSION ON THE TARGET 1

$$\text{If } C < \frac{1}{p_A}, \text{ then } Y_\infty > 0 \text{ a.s.}$$

since

$$\mathbf{1}_{\{Y_\infty=0\}} = \lim_n \mathbb{P}(Y_\infty = 0 \mid \mathcal{F}_n) = 0 \text{ a.s.}$$

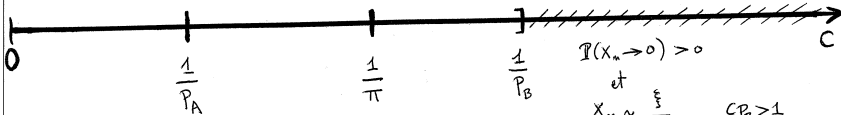
i.e.

$$C < \frac{1}{p_A} \implies 1 - X_n \sim \underbrace{\xi Y_\infty}_{>0} n^{-C p_A} \text{ a.s.}$$

$$P_A > P_B \text{ et } \gamma_m = \frac{C}{C+m}, m \geq 1$$

infaillibilité

faillibilité



convergence vers 1
à vitesse
 $n^{-C\pi}, C\pi < 1$

convergence vers 1 à vitesse $n^{-CP_A}, CP_A > 1$

$\neq \emptyset$ si $P_A > 2P_B$

$\frac{1}{p_A} < C < \frac{1}{\pi}$: a two regime convergence !

▷ On the one hand, by a symmetry argument (coming from fallibility)

$$\sum_n \prod_{k \geq n} (1 - p_A \gamma_k) \asymp \sum_n n^{-C p_A} < +\infty \implies \mathbb{P}(X \sim \zeta n^{-C p_A}) > 0, \zeta > 0 \text{ a.s.}$$

▷ On the other hand, one shows that

$$\mathbb{E}(X_n Y_n | \mathcal{F}_n) \geq X_{n-1} Y_{n-1} \left(1 - p_A \frac{\gamma_n^2}{\prod_{1 \leq k \leq n} (1 - \pi \gamma_k X_{k-1})} \right)$$

so that

$$\begin{aligned} \mathbb{E}(X_\infty Y_\infty) &\geq x(1-x) \prod_{n \geq 1} \left(1 - p_A \frac{\gamma_n^2}{\prod_{1 \leq k \leq n} (1 - \pi \gamma_k)} \right) \\ &\gtrsim x(1-x) \prod_{n \geq 1} \left(1 - \frac{p_A}{n^{2-C\pi}} \right) > 0. \end{aligned}$$

$$\mathbb{E}(\mathbf{1}_{\{X_\infty=1\}}Y_\infty) = \mathbb{E}(X_\infty Y_\infty) > 0$$

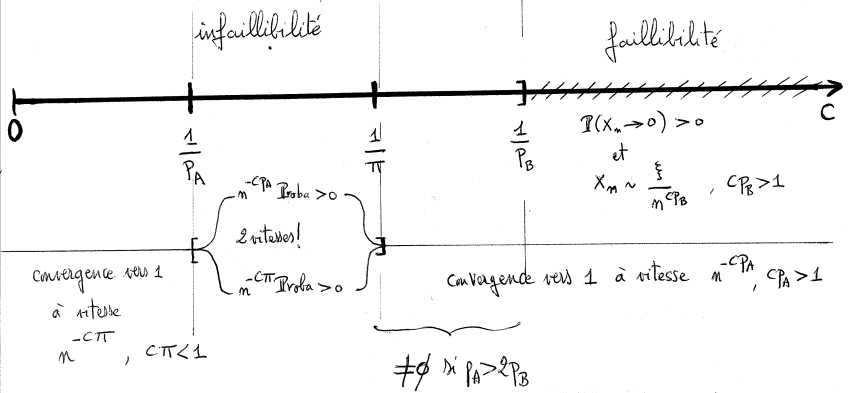
which implies

$$\mathbb{P}(X_\infty = 1, Y_\infty > 0) > 0$$

i.e.

$$\mathbb{P}(X \sim \zeta' n^{-C\pi}) > 0, \quad \zeta' > 0 \text{ a.s.}$$

$$\gamma_m = \frac{C}{C+m}, m \geq 1$$



▷ Blind choice $\equiv C = 1$! Infallibility but (possibly very) poor rate

$$1 - X_n \sim \Xi_x n^{-(p_A - p_B)}$$

What about infallibility **and** good rates?

What about infallibility and good rates?

Lamberton-P. (2008) in *EJP* (2008) introduce

$$LRI \longrightarrow LRP$$

▷ (Still) Focus on the share managed by trader A .

$$X_{n+1} = X_n + \gamma_{n+1} \times \begin{cases} \mathbf{1}_{\{U_{n+1} \leq X_n\} \cap A_{n+1}} (1 - X_n) \\ \mathbf{1}_{\{U_{n+1} > X_n\} \cap B_{n+1}} (-X_n) \end{cases} + \gamma_{n+1} \rho_{n+1} \times \begin{cases} \mathbf{1}_{\{U_{n+1} \leq X_n\} \cap A_{n+1}^c} (-X_n) \\ \mathbf{1}_{\{U_{n+1} > X_n\} \cap B_{n+1}^c} (1 - X_n) \end{cases} \in (0, 1)$$

where $X_0 = x \in (0, 1]$.

- γ_n reward parameter.
- $\gamma_n \rho_n$ penalty parameter.

▷ **THEOREM.** If $\sum_n e^{-\frac{\vartheta}{\gamma_n}} < +\infty$ and

$$\sup_n \frac{\gamma_n}{\rho_n} < +\infty, \quad \sum_n \gamma_n \rho_n = +\infty \quad \text{and} \quad \rho_n \rightarrow 0$$

then

$$(p_A > p_B) \implies (X_n \xrightarrow{a.s.} 1)$$

▷ **SKETCH OF PROOF.**

$$X_{n+1} = X_n + \gamma_{n+1} \pi X_n (1 - X_n) + \gamma_{n+1} \rho_{n+1} \kappa(X_n) + \gamma_{n+1} \Delta M_{n+1}$$

where κ is a “noiseless repeller”

$$\kappa(x) = -(1 - p_A)x^2 + (1 - p_B)(1 - x)^2, \quad \kappa(0) = (1 - p_B) > 0.$$

- The *ODE* method implies $a.s. \lim_n X_n \in \{0, 1\}$ as before.
- The *noiseless repeller* effect:

$$X_n \geq x + \sum_{k=1}^n \gamma_k \rho_k \kappa(X_{k-1}) + \sum_{k=1}^n \gamma_k \Delta M_k$$

and

$$\left\| \sum_{k=1}^n \gamma_k \Delta M_k \right\|_2^2 \leq C \sum_{k=1}^n \gamma_k^2 \leq C' \sup_n \frac{\gamma_n}{\rho_n} \times \sum_{k=1}^n \gamma_k \rho_k$$

so that

$$\frac{\sum_{k=1}^n \gamma_k \Delta M_k}{\sum_{k=1}^n \gamma_k \rho_k} \xrightarrow{L^2(\mathbb{P})} 0 \implies a.s. \limsup_n \frac{\sum_{k=1}^n \gamma_k \Delta M_k}{\sum_{k=1}^n \gamma_k \rho_k} \geq 0.$$

Consequently, **on the event** $\{X_n \rightarrow 0\}$,

$$\limsup_n \frac{X_n}{\sum_{k=1}^n \gamma_k \rho_k} \geq \limsup_n \frac{\sum_{k=1}^n \gamma_k \rho_k \kappa(X_{k-1})}{\sum_{k=1}^n \gamma_k \rho_k} = \kappa(0) > 0 \quad a.s.$$

▷ **Theorem 1** (a) “Cvgce assumptions” + $\rho_n - \rho_{n-1} = o(\rho_n \gamma_n)$.

$$Y_n := \frac{1 - X_n}{\rho_n} \xrightarrow{\mathbb{P}} \frac{1 - p_A}{\pi}.$$

(b) $\gamma_n = c_1 n^{-a}$, $\rho_n = c_2 n^{-b}$, $a + b < 1$, $b < a$, then

$$Y_n \xrightarrow{a.s.} \frac{1 - p_A}{\pi}$$

Key : Bennett's Inequality for bounded martingale increments (see *e.g.* Meyer).

▷ THEOREM 2 “Cvgce assumptions” + $\gamma_n = g\rho_n$, $g > 0$.

(a) A pseudo-CLT:

$$Y_n \xrightarrow{\mathcal{L}} \nu,$$

ν invariant distribution of a càdlàg (mean-reverting) Markov process $(Y_t)_{t \geq 0}$

$$dY_t = (1 - p_A - \pi Y_t)dt + dN_{\mu \int_0^t Y_s ds}, \quad \mu = \frac{p_B}{g}, \quad N \text{ standard Poisson process.}$$

(b) Absolute continuity: the distribution $\nu = \varphi \cdot \lambda_d$, φ solution to a delay ODE.

Practical recommendations: Best rate with

$$\gamma_n = \rho_n = n^{-\frac{1}{2}} \quad (\text{Weak}) \text{ Rate of convergence} = \sqrt{n}.$$

- D. Lamberton, G. Pagès, P. Tarrès, When can the two-armed bandit algorithm be trusted ? *The Annals of Applied Probability*, **14**(3), 1424-1454, 2004.
- D. Lamberton, G. Pagès, How fast is the bandit? *Stochastic Analysis and Applications*, **26**:603-623, 2008.
- D. Lamberton, G. Pagès, A penalized bandit algorithm, *Electronic Journal of Probability*, **13**:341-373, Mai 2008.

101 QUIZZ QUI BANQUENT

Gilles Pagès

Illustré par Nicolas Dahan

Mathématiques et finance
sont-elles indépendantes ?



La plupart des questions
révisées dans ce volume
proviennent du site :
www.maths-6.com,
le site de l'emploi et de
la formation en finance
de marché.

Vous avez surpris votre fils à allumer des cierges de toutes les tailles devant la télévision pour mesurer la durée des matchs de football juste après avoir observé, incrédule, votre fille nouer au hasard les extrémités de ses spaghettis *al dente*, formant ainsi dans son assiette des cercles qu'elle répertorie avec soin. Rassurez-vous, ces jeunes gens ne sont pas sous la menace d'une crise mystique ou d'anorexie, ils envisagent simplement de se lancer dans la carrière d'analyste en finance quantitative, voire, qui sait, de trader.

Une ambition de ce recueil est de les y aider (un peu) en recensant et résolvant une sélection de 101 « énigmes » proposées lors d'entretiens d'embauche (ou de stage) en finance quantitative dans les plus prestigieux établissements présents sur les places financières européennes.

Autre ambition : que tous les lecteurs y trouvent matière à méditer – et surtout à se torturer les méninges – pour triompher de ce parcours de quizz parsemé d'embûches logiques, de paradoxes probabilistes et d'astuces perverses. Au détour d'un chapitre on y croiera même quelques célébrités.

Professeur à l'université Pierre et Marie Curie/Paris-VI, spécialiste des probabilités, Gilles Pagès y enseigne les mathématiques financières dans le cadre du Master « Probabilités et finance », créé au début des années 1990 par l'université Pierre et Marie Curie et l'école polytechnique, sous l'impulsion de Nicole El Karoui.

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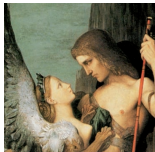


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