Proposition de sujet de thèse

Intitulé français du sujet de thèse proposé :
Résonances en théorie de la diffusion : complétude pour potentiels optiques et gravitationnels

Intitulé en anglais :
Scattering resonances: completeness aspects in optical and gravitational potentials

Unité de recherche :
Institut de Mathématiques de Bourgogne (IMB)

Nom, prénom et courriel du directeur (et co-encadrant) de thèse :
Jaramillo, Jose-Luis (Jose-Luis.Jaramillo@u-bourgogne.fr)

Domaine scientifique principal de la thèse :
Scattering theory, spectral analysis.

Domaine scientifique secondaire de la thèse :
Partial differential equations, classical field theory, spectral geometry.

Description du projet scientifique
Antecedents : The dynamics of open systems is characterized by the leakage of some relevant quantity away from the system. Such non-conservative behavior translates into a description in terms of non-Hermitian operators in the associated (sourceless) wave propagation problem, once purely outgoing boundary conditions are imposed. A key role in the dynamics is played by the discrete set of the corresponding resonant wave solutions, known as resonant states or quasi-normal modes, with characteristic oscillation and time decay rate encoded in their complex scattering resonance frequencies. These are the analogues in open systems to bound eigenstates in conservative situations. Although quasi-normal modes do not represent in general a complete set for the expansion of the relevant fields, they do provide an efficient approach for analysing the fields in certain physical regimes, e.g. at intermediate stages of dynamical relaxation. Applications in optical settings (namely in resonant nano-cavities and in wave-guides) and in gravitational systems (e.g. ringdown modes of black hole spacetimes) have attracted much attention in recent times. This is the setting of the present thesis project.
Objective : The general objective is the study of the completeness properties of scattering resonances of specific potentials appearing in optical and gravitational problems. More concretely, this thesis research aims at assessing the dynamical bounds (in time and space) permitting a controlled approximation of the scattered field in terms of the set of quasi-normal modes. In this general setting, two restricted objectives are attempted: i) modeling of the electro-magnetic scattered field in the vicinity of elongated nano-particles, and ii) analysis of near-zone strong gravitational field in the vicinity of the horizon in a black hole spacetime.
Methodology : the research projects combines mathematical tools from spectral analysis with physical inputs from classical field theory (in the setting of optics and general relativistic problems). A major point is that the optical part of the project stands as part of a close collaboration with the Laboratoroire Interdisciplinaire Carnot de Bourgogne (ICB), in particular aiming at the light manipulation at nanometric scales by means of surface plasmons supported on metallic nano-structures. The quasi-normal mode analysis will provide an approximation tool complementing the numerical and experimental approaches by our physicist partners. Regarding the gravitational part of the research, the spectral analysis tools will be combined with (semi-)Riemannian geometric tools to probe the spacetime curvature structure in the

vicinity of the black hole, in the context of understanding the gravitational wave ringdown of the system. Methodologically, the optical system is meant as a toy-model for the (extremely challenging) gravitational problem. The research methodology is essentially interdisciplinary.

Programme : As a tentative working program sketch, research stages are articulated around the following milestones: i) resolution of the scattering resonance problem in a simplified one-dimensional model of a nano-particle: numerical study; ii) extension of the result to an exactly solvable model and detailed study of the completeness issues: identification of the early and late time bounds; iii) extension of the numerical/analytical previous analysis to more realistic three-dimensional (elongated nano-particle) potentials: comparison with results from the physicists team; iv) extension of the results to the Schwarzschild gravitational potential: assessment of similarities and differences with the optical case; v) expansion of the near-horizon gravitational field in terms of the scattering resonances of the black hole.

Connaissances et compétences requises :

Classical field theory : electromagnetism (gravitational theory advisable, not critical). Advisable: elements of spectral theory and numerical skills. Priority will be given to candidates with a working knowledge in quasi-normal modes, ideally in the optics and/or nanoparticles.