

# Singularities, knots, and mapping class groups in memory of Bernard Perron

Dijon, September 6th to 9th, 2010

## Abstracts

### Invited speakers

**Norbert A'CAMPO**

**Title.** *Finite dimensional representations of the mapping class group from measured foliations and webs.*

**Abstract.** *The spaces of measured  $k$ -webs on a Riemann surface of genus  $g > 1$  define a complex vector bundle of dimension  $(2k - 1)(g - 1)$  over Teichmueller space. We define a flat connexion on this bundle, hence we obtain for each  $k > 0$  a linear representation of the mapping class group. The representations are asymptotically faithful. Further properties are under study, especially the relation(s) with the TQFT-representations.*

**Riccardo BENEDETTI**

**Title.** *Levels of knotting and corank of spatial handlebodies*

**Abstract.** *Starting from the ground-zero level that requires the non-existence of any planar spine, one defines a few increasingly levels of knotting of (genus 2) handlebodies in  $S^3$ , in terms of the non-existence of spines verifying decreasingly special properties. One shows that every level is non empty, the levels are strictly increasing, and a handlebody does not belong to the top level iff the complementary domain has maximal corank. To do this, one uses some instance of rather recent “quandle invariants” of spatial handlebodies (Ishii, 2009), as well one recognizes a few “old-fashioned” results of 3-dimensional topology.*

*I like to believe that Bernard would have found this stuff amusing.*

**Michel BOILEAU**

**Title.** *Commensurability of hyperbolic knot complements*

**Abstract.** *We study commensurability classes of hyperbolic knot complements in  $S^3$ . In the generic case of knots without hidden symmetries, we show that knots complements which are commensurable are cyclically commensurable, and that there are at most 3 distinct hyperbolic knot complements in the same commensurability class. Moreover if two hyperbolic knot without hidden symmetries have commensurable complements, then they are fibered with the same genus and chiral. Characterization of commensurability classes of complements of periodic knots without hidden symmetries is also given.*

*(Travail en collaboration avec S. Boyer, R. Cedanu et G. Walsh)*

## François LAUDENBACH

**Title.** *Open books and twisted open books in dimension 3. Application to singular codimension-one foliations*

**Abstract.** *The lecture will mainly refer to a joint work with Gaël Meigniez. The first part is devoted to popularize ideas of E. Giroux about the way of building an open book from a Morse function. Despite the binding (which is a removable singularity), an open book has a trivial normal bundle. A twisted open book is a similar object with a twisted normal bundle; the complement of the binding has a Seifert fibration over  $[-1, +1]$ .*

*We consider singular 2-foliations on  $M^3$  whose singularities are singularities of functions (ie.  $\Gamma$ -structures in Haefliger's sense). Any such foliation whose normal bundle embeds into the tangent bundle to  $M$  is concordant to a non-singular foliation "carried" by an open book or a twisted open book. This statement contains Thurston's regularization theorem in dimension 3 and its proof does not use any difficult theorem on  $\text{Diff}_c^\infty(\mathbb{R})$ .*

## LE DUNG TRANG

**Title.** *La Balançoire*

**Abstract.** *Dans cette conférence je vais rappeler une technique de démonstration que j'ai utilisée avec Bernard Perron.*

## Gwénaél MASSUYEAU

**Title.** *Equivalence relations on homology cylinders defined by the lower central series of the Torelli group.*

**Abstract.** *Let  $k$  be a positive integer. Two 3-manifolds  $M$  and  $M'$  are said to be " $Y_k$ -equivalent" if  $M'$  can be obtained from  $M$  by "twisting" an embedded surface  $E$  of  $M$  with an element of the  $k$ -th term of the lower central series of the Torelli group of  $E$ . In this talk, we shall consider the  $Y_k$ -equivalence relation among homology cylinders over a given surface  $S$ . We will review some of its general properties and its connections with the Goussarov-Habiro theory of finite-type invariants. Next, we will see that the  $Y_3$ -equivalence relation is characterized by three invariants of homology cylinders: the action on the third nilpotent quotient of the fundamental group of  $S$ , the quadratic part of (a relative version of) the Alexander polynomial and the (core of the) Casson invariant. This characterization is joint work with J.-B. Meilhan.*

## Dale ROLFSEN

**Title.** *Eigenvalues, fibred knots and ordered groups*

**Abstract.** *Several years ago, James Howie and Hamish Short showed that all knot groups can be given a left-invariant ordering. This raised the question: which knot groups enjoy a two-sided invariant ordering? This query came up while I was visiting both Marseille and Dijon. With Bernard's crucial algebraic insight he and I were able to show that the figure-eight knot group is bi-orderable. The same is true of other fibred knots whose Alexander polynomial has all roots real and positive. That was a consequence of the following algebraic theorem of ours: if  $h$  is an automorphism of the free group  $F$ , and if the induced automorphism of the abelianization  $F/F'$  has all eigenvalues real and positive, then there is a bi-ordering of  $F$  which is invariant under  $h$ . Later we showed the same result for surface groups in place of  $F$ , which in turn gave applications to closed 3-manifolds which fibre over the circle. Again, in that second LMS paper, all the hard algebra was worked out by Bernard.*

*Quite recently, Adam Clay and the speaker have found a sort of converse, which holds for any finitely generated nontrivial group  $G$ . If  $G$  has a bi-ordering which is invariant under an automorphism  $h$  of  $G$ , then the induced map on the (rational) first homology of  $G$  must have at least one positive eigenvalue. A topological consequence is that many fibred knots do NOT have bi-ordered*

knot group, because their Alexander polynomial has no positive real roots. We also show that if nontrivial surgery on a nontrivial knot produces an  $L$ -space (as defined by Ozsvath - Szabo), then the knot's group cannot be bi-orderable.

## Peter SHALEN

**Title.** *Margulis numbers of hyperbolic 3-manifolds*

**Abstract.** *Let  $M = \mathbf{H}^3/\Gamma$  be an orientable hyperbolic 3-manifold. A Margulis number for  $M$  is a positive real number  $\mu$  with the following property: for any  $P \in \mathbf{H}^3$ , if  $x$  and  $y$  are elements of  $\Gamma$  such that  $\max(d(P, x \cdot P), d(P, y \cdot P)) < \mu$ , then  $x$  and  $y$  commute. Culler and I have recently shown that if  $M$  is homeomorphic to the interior of a Haken manifold  $N$  then 0.286 is a Margulis number for  $M$ , and if in addition  $N$  has non-empty boundary then 0.292 is a Margulis number for  $M$ . I have combined this with character-variety techniques to show that all but at most finitely many closed, orientable hyperbolic 3-manifolds admit 0.292 as a Margulis number. Using different arguments, I have shown that if  $0 < \lambda < (\log 3)/2$ , there is an explicit bound on the rank of  $\pi_1(M)$  for all closed, orientable hyperbolic 3-manifolds  $M$  that do not admit  $\lambda$  as a Margulis number. The proof actually gives a bound on the minimal index of a rank-2 subgroup of  $\pi_1(M)$ . Combining the proof of this last result with earlier work of mine, I can give an explicit bound, in terms of the degree of a number field  $K$ , on the number of  $\mathbf{Z}_6$ -homology 3-spheres with trace field  $K$  that do not admit 0.183 as a Margulis number.*

## Claude WEBER

**Title.** *Corps avec anses et fibre de Milnor*

**Abstract.** *Je ferai une introduction au papier de Le-Perron dans lequel ils démontrent que la fibre de Milnor est toujours un corps avec anses. Le cas est plus compliqué lorsque la dimension réelle de la fibre est 4. Je ferai une sorte de survey sur les corps avec anses en général (Wall - Haefliger) et sur la dimension 4 en particulier. S'il reste du temps je parlerai des diverses définitions du type topologique d'une singularité – qui, finalement, sont toutes équivalentes –. À nouveau il y a une importante contribution de Bernard Perron dans un cas difficile.*

## Other speakers

### Mark BAKER

**Title.** *Arithmetic knots in spherical 3-manifolds*

**Abstract.** *It is well known that the figure eight knot,  $k$ , is the only arithmetic knot in  $S^3$ . Indeed,  $S^3 \setminus k$  is a 12-fold cover of the Bianchi orbifold  $H^3/PSL(2, O_3)$ . Generalizing, we prove the following*

*Theorem :* *If  $M$  is a closed, orientable, spherical 3-manifold and  $K$  a knot in  $M$  such that there exists a finite cover  $M \setminus K \rightarrow H^3/PSL(2, O_d)$ , then necessarily  $d = 3$ .*

*The proof involves analyzing Dehn fillings on the Bianchi orbifolds  $H^3/PSL(2, O_d)$ . (This is joint with Alan Reid).*

### Paolo BELLINGERI

**Title.** *Representations for surface braid groups and lower central series*

**Abstract.** *“Lawrence representations of the braid group  $B_n$  can be constructed considering particular exact sequences and lower central series of braid groups”. We will explain briefly the previous statement and we will explain how lower central series of surface braid groups can turn out to be useful for getting representations for surface braids. In particular we will describe obstructions and the notion of extension of representation recently introduced and studied by An and Ko using a*

complete algebraical framework. The talk is a survey on two joint works with V. Bardakov and with E. Godelle and J. Guaschi

## Christian BLANCHET

**Title.** *On the core of Casson invariant*

**Abstract.** *We start with Bernard Perron's work on Casson invariant. Following Kristell Dequidt Picot Phd thesis we extend Meyer and intersection 2-cocycles from the mapping classgroup to the cobordism group of homology cylinders. The intersection cocycle a priori depends on a choice of homotopy class of vector field that we will fix. We then define Morita framing and revisit Casson invariant.*

## Pierre DERBEZ

**Title.** *Volume of representation of 3-manifold groups*

**Abstract.** *We will state some relations between the volume of representations of 3-manifold groups and some invariants given by the gauge theory. Next we will give some applications.*

## François DIGNE

**Title.** *Structures de Garside pour des groupes d'Artin-Tits de type affine*

**Abstract.** *Les groupes d'Artin-Tits de type sphérique ont deux structures de Garside, classique et duale. La structure classique disparaît pour les autres groupes d'Artin-Tits, mais, au moins, pour les groupes affines de type A ou C il existe une structure duale. Ceci s'appuie sur leur interprétation comme groupes de tresses et sur la combinatoire des partitions sans croisements.*

## Roger FENN

**Title.** *Invariants of knotted surfaces*

**Abstract.** *There is a well known construction of a smooth torus in  $R^4$  from a welded knot. It turns out that this is surjective but not injective either in the DIFF or TOP categories. This talk will look at some invariants to distinguish tori and other surfaces in  $R^4$ .*

## Thomas FIEDLER

**Title.** *Invariants for knots and string links via one parameter knot theory*

**Abstract.** *We construct new knot invariants by combining the representation theory of Lie groups with the singularity theory of one parameter families of knots in 3-space. Let  $T$  be an ordered oriented string link of  $n$  components, e.g. a satellite of a framed long knot. The well known HOMFLY-PT invariant  $P_T$  is an element of the Hecke algebra  $H_n$ . Its coefficients are Laurent polynomials of two variables,  $z$  and  $v$ . To each bijection  $\sigma$  of two subsets of cardinal  $k$  of the set  $\{1, \dots, n\}$  we associate a new object: the Hecke groupoid  $H_{n,\sigma}$ . It is generated by the superposition of all basis elements  $e_i$  of  $H_k$  with all basis elements  $e_j$  of  $H_{n-k}$  according to the bijection  $\sigma$ . Here, all crossings between  $e_i$  and  $e_j$  are defined to be virtual. The coefficients in  $H_{n,\sigma}$  are Laurent polynomials of three variables,  $z$ ,  $v$  and a new variable  $y$ . Let  $\mathcal{H}_n$  be the union of all  $H_{n,\sigma}$  for all  $\sigma$ .*

*For each cyclic permutation  $\phi \in S_n$ , each subset  $A \subset \{1, \dots, n\}$  and each natural number  $m$ ,  $1 \leq m \leq n$ , we construct an isotopy invariant  $R_{\phi,A,m}(T) \in \mathcal{H}_n$  of  $T$ . In the special case  $A = \emptyset$ ,  $R_{\phi,A,m}(T)$  does not depend on  $\phi$  and is an element of  $H_{n,\emptyset}$ , (i.e.  $k = 0$ ). It turns out that in this case the absolute term with respect to  $y$  of  $R_{\phi,\emptyset,m}(T)$  coincides with  $P_T$ .*

*The new invariants have lots of properties which distinguish them from the known quantum knot invariants.*

## Louis FUNAR

**Title.** *Groups which are not properly 3-realizable*

**Abstract.** *A finitely presented discrete group is properly 3-realizable if it is the fundamental group of a finite 2-complex whose universal cover is proper homotopy equivalent to a 3-manifold. One-relator groups, non-trivial products 3-manifold groups are properly 3-realizable. We give a conjectural characterization of properly 3-realizable groups in terms of their asymptotic topology and prove it works for a large class of groups. In particular we find the first examples of finitely presented groups which are not properly 3-realizable, which are specific Coxeter groups. This is joint work with F.Lasher and D.Repovs.*

## John GUASCHI

**Title.** *Embeddings of the braid groups of covering spaces, classification of the finite subgroups of the braid groups of the real projective plane, and linearity of braid and mapping class groups of low-genus surfaces*

**Abstract.** *Let  $M$  be a compact, connected surface, possibly with a finite set of points removed from its interior. Let  $d, n \in \mathbb{N}$ , and let  $\tilde{M}$  be a  $d$ -fold regular covering space of  $M$ . We show that the covering map induces an embedding of the  $n$ -string braid group of  $M$  in the  $dn$ -string braid group of  $\tilde{M}$ , and we give several applications of this result. First, we classify the finite subgroups of the  $n$ -string braid group of the real projective plane, from which we deduce an alternative proof of the classification of the finite subgroups of the mapping class group of the  $n$ -punctured real projective plane due to Bujalance, Cirre and Gamboa. Secondly, using the linearity of  $B_n$  due to Bigelow and Krammer, we show that the braid and mapping class groups of compact, connected surfaces of low genus are linear (joint work with D.L. Gonçalves).*

## Joel HADDLEY

**Title.** *Symmetric Singularities and Complex Hyperbolic Symmetric Groups*

**Abstract.** *An important classical result of singularity theory is the identification of the monodromy groups of simple, isolated singularities with the known Coxeter groups of types  $A$ ,  $D$ ,  $E$ . A generalization of this approach gives a similar correspondence between isolated singularities and groups generated by complex reflections. In this talk, we demonstrate how an exceptional unimodal singularity considered with a symmetry has as its symmetric monodromy group a group generated by complex reflections with hyperbolic signature.*

## Arkadius KALKA

**Title.** *Subgroup conjugacy problem for Garside subgroups of Garside groups*

**Abstract.** *First, we give a short overview on the Garside-theoretic solution of the conjugacy problem in braid groups, which is by far not meant to be complete. A more general problem is the subgroup conjugacy problem for  $H \leq G$ . Given two elements  $x, y \in G$ , and a subgroup  $H \leq G$ , decide whether  $x$  and  $y$  are conjugated by an element in  $H$ . We solve the subgroup conjugacy problem for parabolic subgroups and Garside subgroups of a Garside group, and we present deterministic algorithms. This solution may be improved by using minimal simple elements. For standard parabolic subgroups of Garside groups we provide effective algorithms for computing minimal simple elements. Both, the notions of parabolic subgroup and of Garside subgroup of a Garside group have been introduced by Eddy Godelle. This is joint work with Eran Liberman and Mina Teicher. Furthermore, we provide solutions for several other subgroup conjugacy problems in braid groups where  $H$  is some other natural subgroup of  $B_n$ , but not parabolic or Garside.*

## Ulrich KOSCHORKE

**Title.** *Coincidences in torus and sphere bundles*

**Abstract.** *Fixed points and coincidences of maps between manifolds (and attempts to remove them by suitable homotopies) lead to geometric obstructions which are intimately related to some deep notions of differential topology (such as e.g. Kervaire invariants). We illustrate the resulting beautiful Nielsen theory in the framework of fiberbundles.*

## Pierre-Vincent KOSELEFF

**Title.** *Polynomial parametrizations of rational knots.*

**Abstract.** *It is known that every (non compact) knot admits a polynomial parametrization. When  $K$  is a rational (or 2-bridge) knot with  $N$  crossings, we describe an explicit parametrization of  $K$  with degrees  $(3, b, c)$  and  $b+c = 3N$ . Joint work with Daniel Pecker Reference : Chebyshev diagrams for two-bridge knots. To appear in Geometriae Dedicata*

## Doug LAFOUNTAIN

**Title.** *Isotopies of links carried by Matsuda branched surfaces*

**Abstract.** *We initiate the study of two related sets of topological objects in the 3-sphere, namely a set of two-component links termed “iterated doubling pairs”, and a set of associated branched surfaces called “Matsuda branched surfaces”. Together these two sets possess a rich internal structure, and allow us to present two theorems that provide a new characterization of topological isotopy of closed braids, as well as a new characterization of transversal isotopy of braids in the 3-sphere endowed with the standard contact structure. This is joint work with Bill Menasco, and builds upon recent seminal work of Hiroshi Matsuda.*

## Michael LONNE

**Title.** *Bifurcation braid monodromy of plane curves*

**Abstract.** *We consider spaces of plane curves in the setting of algebraic geometry and of singularity theory. On one hand there are the complete linear systems, on the other we consider unfolding spaces of bivariate polynomials of Brieskorn-Pham type. For suitable open subspaces we can define the bifurcation braid monodromy taking values in the Zariski resp. Artin braid group. In both cases we give the generators of the image. These results are compared with the corresponding geometric monodromy. It takes values in the mapping class group of braided surfaces. Our final result gives a precise statement about the interdependence of the two monodromy maps. Our study concludes with some implication with regard to the unfaithfulness of the geometric monodromy and the - yet unexploited - knotted geometric monodromy, which takes the ambient space into account.*

## Miguel MALDONADO

**Title.** *The reduced mapping class groups of non-orientable surfaces*

**Abstract.** *Let  $M$  be a closed surface. The group of self-diffeomorphisms of  $M$  acts transitively on the (unordered) configuration space of  $M$ . This action is used to construct a space with fundamental group isomorphic to the reduced mapping class group of  $M$ , denoted by  $\hat{\Gamma}^k(M)$ . This group turns to fit into a short exact sequence of the form  $1 \rightarrow \hat{\Gamma}^k(M) \rightarrow \Gamma^k(M) \rightarrow \Gamma(M) \rightarrow 1$ , with  $\Gamma^k(M)$  the mapping class group with punctures and  $\Gamma(M)$  the mapping class group. In the talk I will give the definition of the reduced mapping class group and present the constructions mentioned above for the case when  $M$  is the projective plane and the Klein bottle.*

## Ivan MARIN

**Title.** *Cubic hecke algebras have characteristic 2*

**Abstract.** *(joint with Marc Cabanes) We investigate the cubic Hecke algebras defined by L. Funar and P. Bellingheri at a natural standard parameter and show that, except in characteristic 2, they collapse for a sufficiently high number of strands.*

## Ricardo MARTINS

**Title.** *The global behavior of the Ricci flow of left invariant metrics on flag manifolds.*

**Abstract.** *We study the global behavior of the Ricci flow equation using methods of the singularity theory and dynamical systems. We present results for homogeneous manifolds with two isotropy summands and for the full flag manifold  $SU(3)/T$ .*

## Gregor MASBAUM

**Title.** *How to approximate quantum representations of mapping class groups by finite groups*

**Abstract.** *The Witten-Reshetikhin-Turaev TQFT-invariants of 3-manifolds give rise to finite-dimensional representations of mapping classgroups of surfaces. I will show how to approximate these representations by representations into finite groups, using the theory of Integral TQFT developed in joint work with P. Gilmer.*

## Hiroshi MATSUDA

**Title.** *An extension of Burau representation, and a deformation of Alexander polynomial*

**Abstract.** *We define an extension of Burau representation of the braid groups. Following the construction of Alexander polynomial of knots from Burau representation, we will construct a knot invariant from our extension of Burau representation.*

## Jean-Francois MATTEI

**Title.** *Mapping Class Group d'un germe de courbe holomorphe plane singulière et classification de feuilletages singuliers marqués*

**Abstract.** *Il s'agit de travaux en collaboration avec David Marin de l'Université Autonome de Barcelone. Nous prouvons que toute conjugaison topologique entre germes de courbes holomorphes planes est "homotope" à une conjugaison qui s'étend aux diviseurs exceptionnels des désingularisations minimales de ces courbes. Grâce à ces résultats nous donnons une présentation explicite du Mapping Class Group d'un germe de courbe singulière. Ces techniques nous permettent de définir une notion de marquage de singularités de feuilletage holomorphes qui intervient de manière essentielle dans la construction d'un invariant topologique complet.*

## Daniel PECKER

**Title.** *Chebyshev knots and billiard knots*

**Abstract.** *Lissajous knots are identical with generic billiard knots in a cube. Chebyshev knots are polynomial analogues of Lissajous knots, they correspond to billiard trajectories in a cube, entering and living through a corner. They are plate closure of billiard braids. As an example, we obtain polynomial parametrizations of the torus knots  $T(2, 2n + 1)$  of degrees  $(3, 3n + 1, 3n + 2)$ . We conjecture that these degrees are lexicographically minimal. Joint work with P.-V. Koseleff  
Reference : Chebyshev knots, to appear in JKTR.*

## Carlo PETRONIO

**Title.** *Complexity of links*

**Abstract.** *I will describe an extension of Matveev's theory of complexity from 3-manifolds to pairs (3-manifold, link), showing that most of (but not all) the main features of this theory generalize to this broadened context. In particular, I will show that the behavior of link complexity under connected sum is not always strictly additive when the connected sum is performed along the link. I will then describe a conjectural formula for the complexity of torus knots in lens spaces. Last, I will show how the theory of complexity can be used as a basis for a specialization to manifold-link pairs of the Turaev-Viro invariants, and I will state some results concerning these specialized invariants, proving their independence from previously known ones.*

## Yves STALDER

**Title.** *Actions propres des produits en couronne*

**Abstract.** *On dit qu'un groupe (disons dénombrable) possède la propriété de Haagerup s'il admet une action métriquement propre (par isométries affines) sur un espace de Hilbert. Après avoir brièvement rappelé l'intérêt de cette propriété, je raconterai un résultat obtenu en commun avec Y. de Cornulier et A. Valette: si  $G$  et  $H$  possèdent la propriété de Haagerup, alors leur produit en couronne aussi.*

## David TROTMAN

**Title.** *Sur les problèmes à  $\mu$  constant chers à Bernard Perron*

**Abstract.** *Une discussion sur le statut actuel des célèbres problèmes à  $\mu$  constant dont il a souvent été question pendant mes conversations avec Bernard Perron, à Orsay, Dijon, Heraklion et Marseille de 1982 aux années 2000. Je'expliquerai quelques progrès récents suggérant de nouvelles pistes, aussi bien topologiques que algébriques.*

## Vladimir VERCHININE

**Title.** *Brunnian Braids on Surfaces*

**Abstract.** *A Brunnian braid means a braid that becomes trivial after removing any one of its strands. A typical example of 3-strand Brunnian braid on a disk is the braid given by the expression  $(\sigma_1^{-1}\sigma_2)^3$ , where  $\sigma_1$  and  $\sigma_2$  are the standard braid generators. The closure of this braid gives the Borromean rings. We consider Brunnian braids on an arbitrary surface. In the main theorem Brunnian braids are characterized algebraically on all surfaces except sphere and projective plane; in the last two cases homotopy groups of 2-sphere are involved in the characterization. This is a joint work with V. G. Bardakov, R. Mikhailov and Jie Wu (arXiv:0909.3387).*

## Yuichi YAMADA

**Title.** *Divide knot presentation of Berge's knots of lens space surgery*

**Abstract.** *Every knot in Berge's list of lens space surgery (ie, knots yielding a lens space by Dehn surgery) is presented as a divide knot. In generic case, such a knot is presented by a billiard plane curve in "L-shaped" region, and the area of the pool is equal to the surgery coefficient. In this research, Couture-Perron's formula works very helpfully.*